Distributed Optimal Consensus in Nonlinear Multi-agent Systems

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INTRODUCTION

Thanks to advances in computing, communication, and sensor and actuator technology, multi-agent cooperative missions have become more implementable. Consensus problem is fundamental to cooperative missions. Consensus is that a group of agents interacting through a communication network reaches an agreement on an agreed value by only knowing the information of their neighbors in a finite or infinite time.

Recently, cooperative optimal control has attracted many researchers. It has applications in motion coordination of multi-agent systems. The optimal consensus problem with convex cost functions for nonlinear dynamic networks has less been considered by the researcher. That is why we are inspired to solve this problem.

We propose an algorithm by which agents try to optimize a team cost function by knowing only their own local cost function, that is a function of their local states and control input, and cooperatively reach consensus in a dynamic network with inherent nonlinear first-order dynamics.

ADVANTAGES OF OPTIMAL CONSENSUS

- Stability Criteria Improvement
- Robustness to Unexpected Disturbances
- Energy Conservation in Lengthened Cooperative Missions

HIGHLIGHTS

- Proposing a continuous-time algorithm to find the solution to distributed optimal consensus problems over networks with nonlinear dynamics
- Seeking optimal values of a consensus protocols instead of finding raw optimal control law
- No Need to Supervisory Control Center

PROBLEM FORMULATION

N agents with continuous-time nonlinear dynamics as

$$\dot{x}_i(t) = f\left(x_i(t), t\right) + u_i(t) \qquad i = 1,\cdots,N$$

$x_i(t)$: Position of agents
$u_i(t)$: Control Input

Distributed optimal consensus problem for the MAS

$$\min_{u_i} \sum_{i=1}^{N} J_i(x_i, u_i)$$

$$s.t.\begin{cases} x_1 = x_2 = \ldots = x_N,\ x_i \in \mathbb{R} \\ \dot{x}_i = f(x_i,t) + u_i \quad i = 1,\cdots,N \end{cases}$$

Assumptions.

- J_i are convex con coercive on their domain.
- The function f : continuous and Lipschitz condition

$$f(0,t) = 0 \quad \forall\, t \geq 0$$

CONSENSUS PROTOCOL

Theorem : Suppose the fixed graph G with Laplacian matrix L has a directed spanning tree and Assumptions hold. Then, consensus of the MAS will be achieved, if the following hold,

1) $$u_i = k \sum_{j \in N_i} a_{ij}(x_i - x_j)$$

2) $$lH^T H + kH^T HL \leq 0$$

where

$$H = \begin{bmatrix} 1 & -1 & 0 & \ldots & 0 \\ 0 & 1 & -1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & 0 & 0 & \ldots & 1 \end{bmatrix}$$

We shall find the optimal values for the parameter k in the above protocol in a distributed fashion.

RESULTS

Optimization Problem with considering the protocol

$$\min_{\substack{k_i \\ i=1,\ldots,N}} \sum_{i=1}^{N} J_i(x_i, x_{j\in N_i}, k_i)$$

$$s.t.\begin{cases} \dot{x}_i = f(x_i,t) + k_i \sum\limits_{j\in N_l}^{N} a_{ij}(x_l - x_j) \\ lH^T H + k_i H^T HL \leq 0 \\ k_1 = k_2 = \cdots = k_N. \end{cases}$$

Updating Rule

$$\dot{k}_i(t) = -\rho(t)\frac{dJ_i}{dk_i} - \sum_{i=1}^{N} a_{ij}(k_i(t) - k_j(t)) + P_\kappa(k_i(t)) - k_i(t)$$

$\rho(t)$ is a time-varying scalar factor.

The above dynamics to update the parameter ki in any agent consists of three different parts:

1) $\rho(t)\dfrac{dJ_i}{dk_i}$ to yield the optimal solution with respect to the cost function .

2) $\sum\limits_{i=1}^{N} a_{ij}(k_i(t) - k_j(t))$ is to provide for consensus on the estimates.

3) Projection term $P_\kappa(k_i(t)) - k_i(t)$ is to lead the optimal solution to the constraint set .

Convergence Analysis Consists of

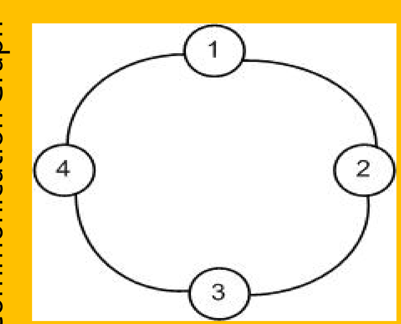
- Existence of Solution
- Constraint Set Convergence
- Optimal Set Convergence
- Consensus Analysis

Computing Total Derivative $\dfrac{dJ_i}{dk_i}$

$$\frac{dJ_1}{dk_1} = \frac{\partial J_1}{\partial k_1} + \frac{\partial J_1}{\partial x_1} z_1 + \frac{\partial J_1}{\partial x_2} z_2 + \frac{\partial J_1}{\partial x_4} z_3$$

where $z_1 := \dfrac{\partial x_1}{\partial k_1}$ $z_2 := \dfrac{\partial x_2}{\partial k_1}$ $z_3 := \dfrac{\partial x_4}{\partial k_1}$

Communication Graph



SIMULATION RESULTS

A network of 4 agents with nonlinear dynamics as

Agents Dynamics $\dot{x}_i = -\sin(x_i) + u_i, \quad x_i, u_i \in \mathbb{R}, \qquad i = 1,\cdots,4,$

Aggregate Cost Function $J(\mathbf{x}) = \mathbf{x}^T\mathbf{x} + \mathbf{u}^T\mathbf{u}$

Adjacency Matrix $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

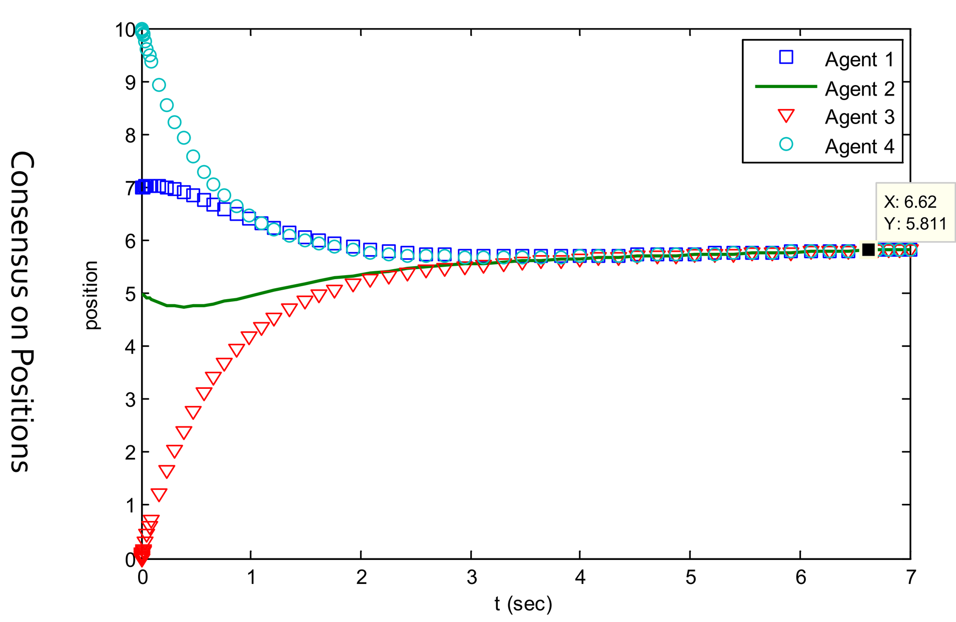
Solution:

$$\min_{\substack{k_i \\ k_i \leq -0.5 \\ i=1,\ldots,4}} \sum_{i=1}^{4} J_i(x_i, x_{j\in N_i}, k_i)$$

$$J_1 = x_1^2 + k_1^2(x_1 - x_4)^2 + k_1^2(x_1 - x_2)^2 \qquad J_2 = x_2^2 + k_2^2(x_2 - x_3)^2 + k_2^2(x_2 - x_1)^2$$

$$J_3 = x_3^2 + k_3^2(x_3 - x_4)^2 + k_3^2(x_3 - x_2)^2 \qquad J_4 = x_4^2 + k_4^2(x_4 - x_1)^2 + k_4^2(x_4 - x_3)^2$$

Consensus on Positions



Optimization Trend