In The Name Of Allah

A Distributed Approach for Solving Multi-Objective Optimal Power Flow

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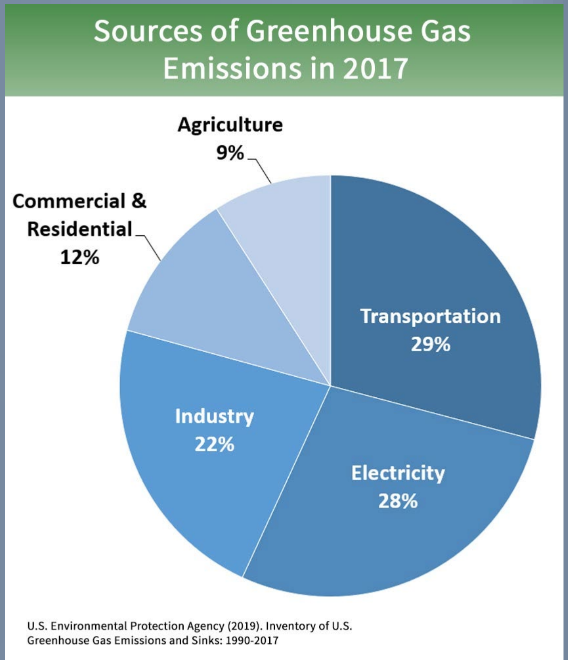
Multi Objective Optimal Power Flow (MOOPF)

- **Optimal Power Flow :** An optimal solution of bus voltage magnitudes and phase angles that minimizes a given generation cost, subject to the power balance at each bus and operational constraints usually include voltage magnitude limits, line-flow limits.
- **Multi-Objective Optimal Power (MOOPF) :** Adverse effect of the electric power industry to the environmental pollution is a matter of concern, so, demanding a great deal of serious actions toward reducing pollution.
- **Greenhouse Gas Emissions by Economic Sector in 2017**



Multi Objective Optimal Power (MOOPF)

❑ MOOPF formulation :

❖ Objective functions

1. Minimization of fuel cost (FC)
2. Minimization of emission objective (FE)
3. Minimization of real power losses (FL)

❖ Constraints

- **Equality constraints**

$$V_l I_l^* = \left(P_l^g - P_l^d\right) + \left(Q_l^g - Q_l^d\right) i \quad ; \forall\, l \in \mathcal{N}$$

$$V_k I_k^* = -P_k^d - Q_k^d\, i \quad ; \quad \forall k \in \mathcal{G}$$

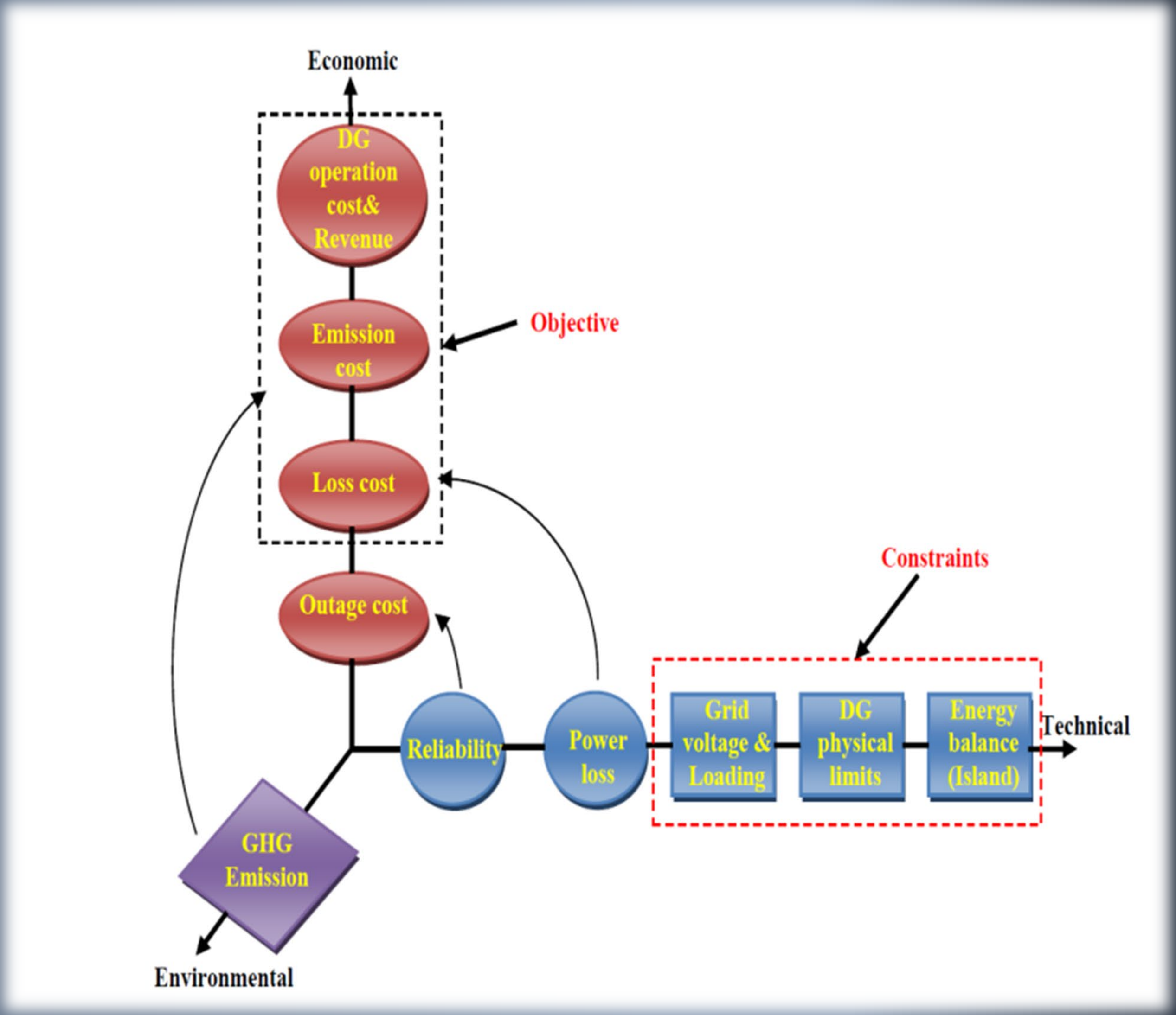
- **Inequality constraints**

$$P_k^{min} \leq P_k^g \leq P_k^{max} \quad ; \quad \forall k \in \mathcal{G}$$

$$Q_k^{min} \leq Q_k^g \leq Q_k^{max} \quad ; \quad \forall k \in \mathcal{G}$$

$$V_l^{min} \leq |V_l| \leq V_l^{max} \quad ; \quad \forall l \in \mathcal{N}$$

$$|S_{kl}| \leq S_{kl}^{max} \quad ; \quad \forall (k,l) \in \mathcal{L}$$



MOOPF formulation

❖ MOOPF formulation

$$\underset{P_l^g, Q_l^g \text{ and } V_k}{minimize} \sum_{l \in G} [FC(P_l^g), FE(P_l^g), FL(P_l^g)] \quad (1)$$

Subject to

$$V_l I_l^* = (P_l^g - P_l^d) + (Q_l^g - Q_l^d)\, i \quad ; \quad \forall l \in N \quad (1.a)$$

$$V_k I_k^* = -P_l^d - Q_l^d i \quad ; \quad \forall k \in G \quad (1.b)$$

$$P_k^{min} \le P_k^g \le P_k^{max} \quad ; \quad \forall k \in G \quad (1.c)$$

$$Q_k^{min} \le Q_k^g \le Q_k^{max} \quad ; \quad \forall k \in G \quad (1.d)$$

$$V_l^{min} \le P_k \le |V_l| \quad ; \quad \forall l \in N \quad (1.e)$$

$$|S_{kl}| \le S_{kl}^{max} \quad ; \quad \forall (k,l) \in N \quad (1.f)$$

$$FC(P_l^g) = c_{l2}(P_l^g)^2 + c_{l1}P_l^g + c_{l0}$$

$$FE(P_l^g) = e_{l2}(P_l^g)^2 + e_{l1}P_l^g + e_{l0} + e_{la}e^{e_{lb}P_l^g}$$

$$FL(P_l^g) = \sum_{l \in G} P_l^g - \sum_{l \in N} P_l^d$$

- P_l^g , Q_l^g : active and reactive powers produced by the generator $l \in \mathcal{N}$.
- P_k^d , Q_k^d : active and reactive loads at buses $k \in \mathcal{N}$.

Power System Control Architecture

- **Centralized:** Each agent communicates with a centralized controller that performs computations and sends new commands.
  - This control structure makes the system vulnerable to single point of failure and communication failures, and raises privacy concerns.

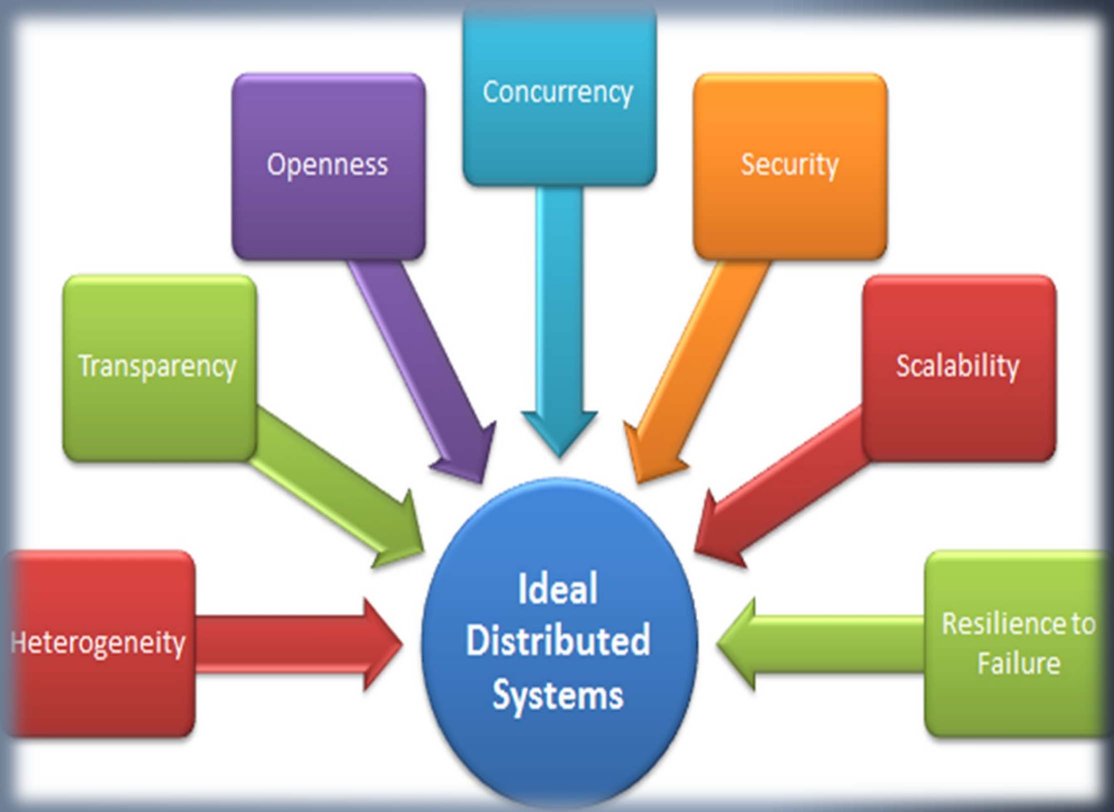
- **Decentralized:** purely local algorithms, i.e., no communication between agents.

- **Hierarchical:** algorithms where computations are done by agents that communicate with other agents at a higher level in a hierarchical structure, eventually leading to a centralized controller.

- **Distributed:** algorithms where each agent communicates with its neighbors, but there is not a centralized controller.

Why Distributed Optimization is Better ?

Distributed algorithms have several potential advantages over centralized approaches. 1) The computing agents only have to share limited amounts of information with a subset of the other agents. This can 2) improve cyber security and 3) reduce the expense of the necessary communication infrastructure. Distributed algorithms also have 4) advantages in robustness with respect to failure of individual agents. Further, with the ability to perform parallel computations, distributed algorithms have the potential to be 5) computationally superior to centralized algorithms, both in terms of solution speed and the maximum problem size that can be addressed. Finally, distributed algorithms also have the potential to respect 6) privacy of data, measurements, cost functions, and constraints, which becomes increasingly important in a distributed generation scenario.



Optimization Dynamical System for MOOPF

❖ Define Optimization Dynamical System for solving MOOPF:

$$\dot{U} = -2a[\boldsymbol{G}(U,\lambda,\gamma,\mu,v) + (e_{n+1}e_{n+1}^T)]\, U \qquad (2.a)$$

$$\dot{\underline{\lambda}}_k = a[P_k^{min} - U^T\boldsymbol{Y}_kU - P_k^d]^+_{\underline{\lambda}_k} \qquad (2.b)$$

$$\dot{\overline{\lambda}}_k = a[U^T\boldsymbol{Y}_kU + P_k^d - P_k^{max}]^+_{\overline{\lambda}_k} \qquad (2.c)$$

$$\dot{\underline{\gamma}}_k = a[Q_k^{min} - U^T\overline{\boldsymbol{Y}}_kU - Q_k^d]^+_{\underline{\gamma}_k} \qquad (2.d)$$

$$\dot{\overline{\gamma}}_k = a[U^T\overline{\boldsymbol{Y}}_kU + Q_k^d - Q_k^{max}]^+_{\overline{\gamma}_k} \qquad (2.e)$$

$$\dot{\underline{\mu}}_k = a\left[\left(V_k^{min}\right)^2 - U^T\boldsymbol{M}_kU\right]^+_{\underline{\mu}_k} \qquad (2.f)$$

$$\dot{\overline{\mu}}_k = a[U^T\boldsymbol{M}_kU - (V_k^{max})^2]^+_{\overline{\mu}_k} \qquad (2.g)$$

$$\dot{v}_{kl} = a[(U^T\boldsymbol{Y}_{kl}U)^2 + (U^T\overline{\boldsymbol{Y}}_{kl}U)^2 - (S_{kl}^{max})^2]^+_{v_k} \qquad (2.h)$$

$$\boldsymbol{G}(U,\lambda,\gamma,\mu) = \sum_{l\epsilon\mathcal{G}}\left[2d_{l2}(U^T\boldsymbol{Y}_lU + P_l^d)\boldsymbol{Y}_l + d_{l1}\boldsymbol{Y}_l + d_{la}.d_{lb}\, e^{d_{lb}\left(U^T\boldsymbol{Y}_lU+P_l^d\right)}\,\boldsymbol{Y}_l\right] + \sum_{k\epsilon\mathcal{N}}(\boldsymbol{\lambda}_k\boldsymbol{Y}_k + \boldsymbol{\gamma}_k\overline{\boldsymbol{Y}}_k + \boldsymbol{\mu}_k\boldsymbol{M}_k) + \sum_{(k,l)\in\mathcal{L}} 2v_{kl}[(U^T\boldsymbol{Y}_{kl}U)\boldsymbol{Y}_{kl} + (U^T\overline{\boldsymbol{Y}}_{kl}U)\overline{\boldsymbol{Y}}_{kl}]$$

❖ Lagrangian saddle-point condition

$$e^{d_{lb}(P_l^{g^{k+1}})} \le \frac{e^{d_{lb}(P_l^{g^k} - P_l^d)}}{d_{lb}(P_l^{g^k} - P_l^d)} \qquad (3)$$

A Distributed Algorithm for Solving MOOPF Problem

Algorithm A: A Distributed Algorithm for Solving MOOPF Problem

1. Define fuel cost, power loss and emission function
2. Given initial conditions Δw_1, Δw_2, Δw_3 and initialize w_1, w_2,$w_3 \leftarrow 0$.
3. **For** (Δw_1 such that $0 : \Delta w_1 : 1$) (Δw_2 such that $0 : \Delta w_1 : 1 - w_1$) **repeat**
    4. **While** until $w_1 + w_2 + w_3 = 1$ **repeat**
    5. Apply the weighting sum method and define MOOPF problem
    6. Define dynamical system (2.a- 2.h) in the form $\dot{U} = F_1(U, \lambda)$ and $\dot{\lambda} = F_2(U, \lambda)$.
    7. Given initial conditions $U_0 \in R^n$, $_0 \in R_p^+$, an error tolerance $\epsilon > 0$, a step size $T > 0$ and initialize k, E, $H \leftarrow 0$.
    8. **While** $||E|| \leq \epsilon$ (until the stopping criterion 1 is satisfied)
    9. compute $(U^{k+1}, \lambda^{k+1}) \leftarrow (U^k, \lambda^k)$
    10. **While** $||H|| \leq \epsilon$ (until the stopping criterion 2 is satisfied)
        10.1. evaluate the vector field (2.a- 2.h) at assign this evaluation to $F = [F_1, F_2]^T$.
        10.2. **For** all i such that $1 \leq i \leq p$ **check**
            If $F_2 \leq 0$ and $\lambda_i^{k+1} \leq 0$
            Then $F_2 \leftarrow 0$ and $\lambda_i^{k+1} \leftarrow 0$
            End for
        10.3. compute $\hat{F} \leftarrow TF - (U^{k+1}, \lambda^{k+1}) + (U^k; \lambda^k)$
        10.4. compute $J \leftarrow$ Jacobian of (2.a- 2.h) at $(U^{k+1}; \lambda^{k+1})$
        10.5. compute $H \leftarrow (TJ - I_{n+p+q})^{-1} - \hat{F}$
        10.6. **For** all i such that $1 \leq i \leq p$ **check**
            If $F_2 = 0$ and $\lambda_i^{k+1} = 0$
            Then $H_{n+i} = 0$
            End for
        10.7. **If** Eq.(3) is satisfied
            Then update $(U^{k+1}, \lambda^{k+1}) \leftarrow (U^{k+1}, \lambda^{k+1}) - H$
            End for
        End while 2
    11. compute $E \leftarrow (U^{k+1}, \lambda^{k+1}) - (U^k, \lambda^k)$ and update $k \leftarrow k + 1$
    End while 1
    12. compute $w_3 \leftarrow w_3 + \Delta w_3$
    End while A
End For A

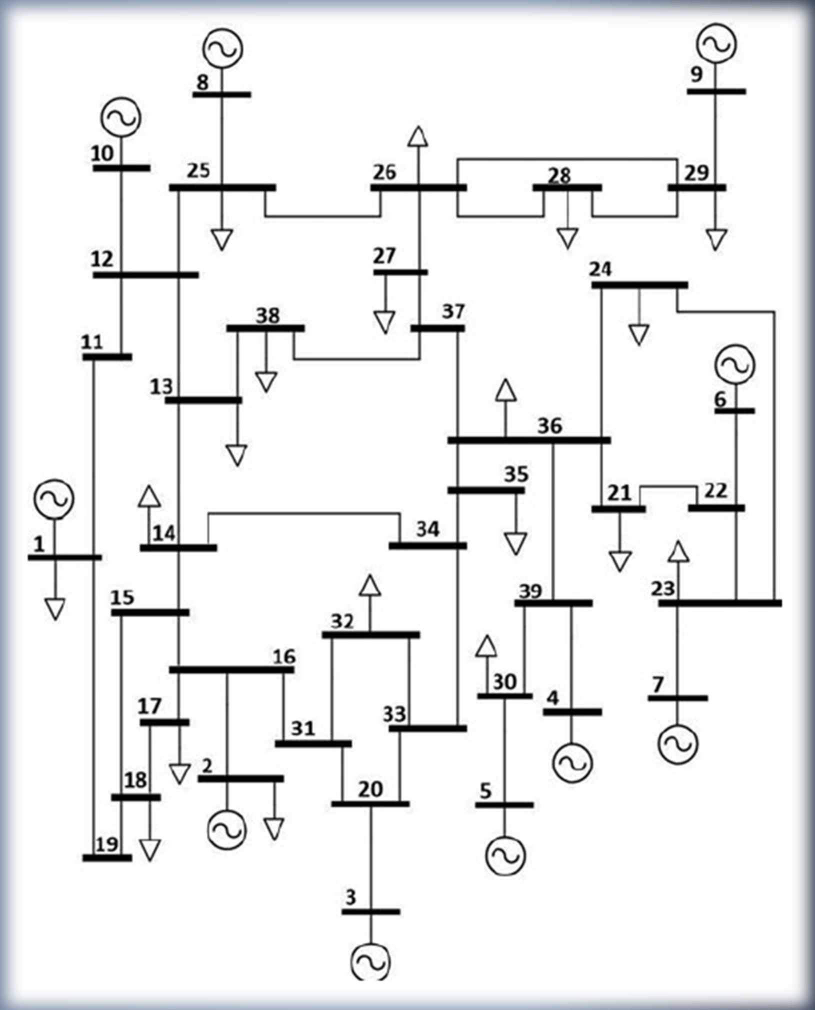
❖ IEEE benchmark power system with 57-bus:

Cost coefficients for generators of IEEE 57-bus system

G.no.	c_{l2}	c_{l1}	c_{l0}
P_{G1}	0.0775795	20	0
P_{G2}	0.01	40	0
P_{G3}	0.25	20	0
P_{G6}	0.01	40	0
P_{G8}	0.0222222	20	0
P_{G9}	0.01	40	0
P_{G12}	0.0322581	20	0

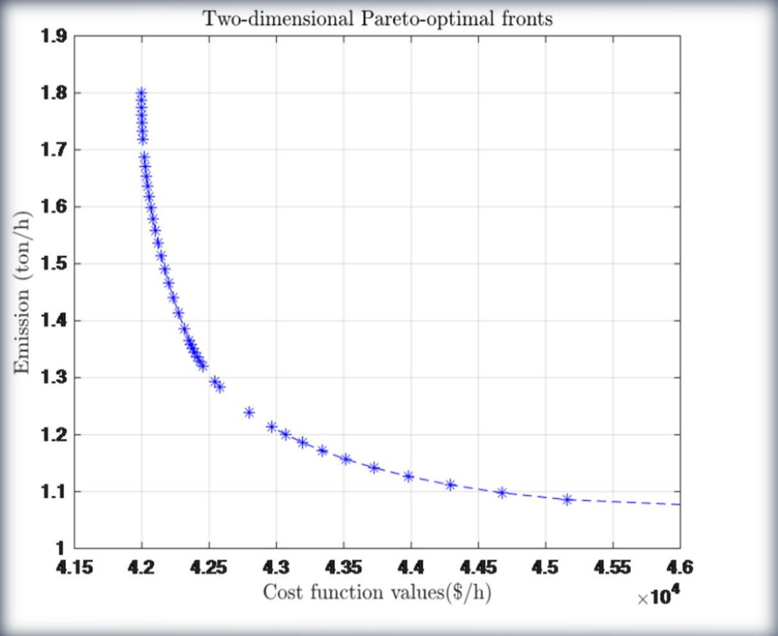
EMISSION COEFFICIENTS OF GENERATORS FOR:57-BUS

G.no.	e_{l2}	e_{l1}	e_{l0}	e_{la}	e_{lb}
P_{G1}	0.060	-0.05	0.040	0.00002	0.5
P_{G2}	0.050	-0.06	0.030	0.0005	1.5
P_{G3}	0.040	-0.05	0.040	0.00001	1.0
P_{G6}	0.035	-0.03	0.035	0.00002	0.5
P_{G8}	0.045	-0.05	0.050	0.00004	2.0
P_{G9}	0.050	-0.04	0.045	0.00001	2.0
P_{G12}	0.050	-0.05	0.060	0.00001	1.5

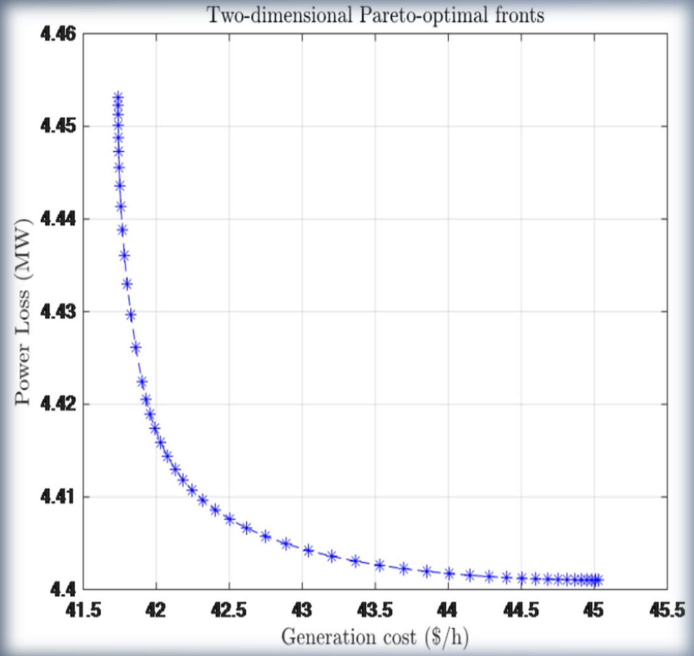


The best Pareto fronts

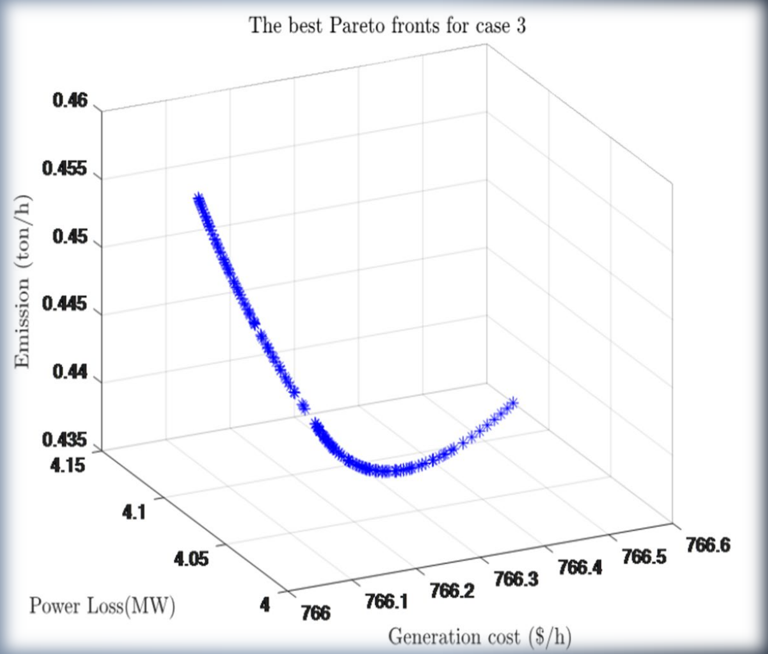
Case 1
Minimize fuel cost and emission



Case 2
Minimize fuel cost and power losses



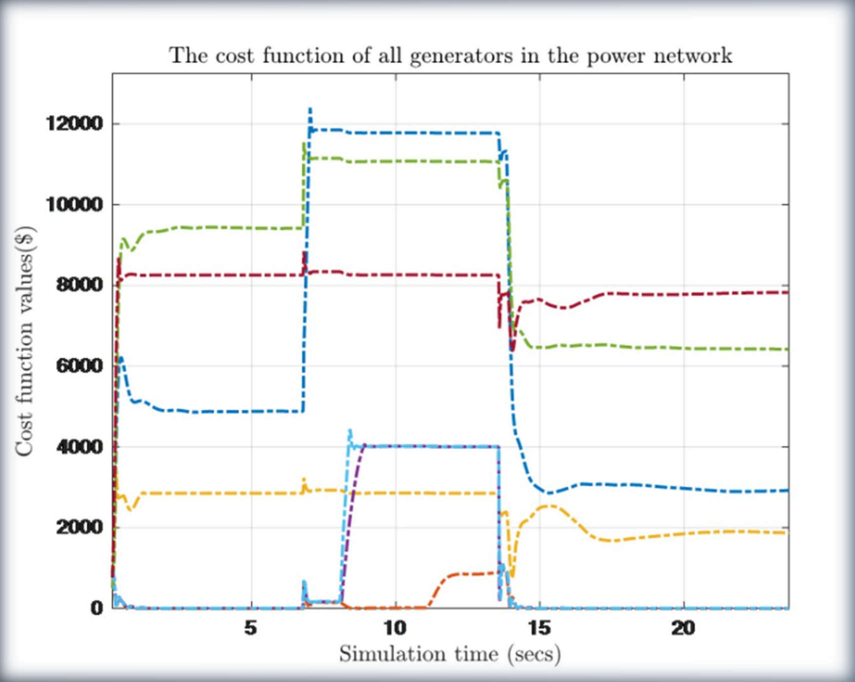
Case 3
Minimize fuel cost, emission function and active power losses.



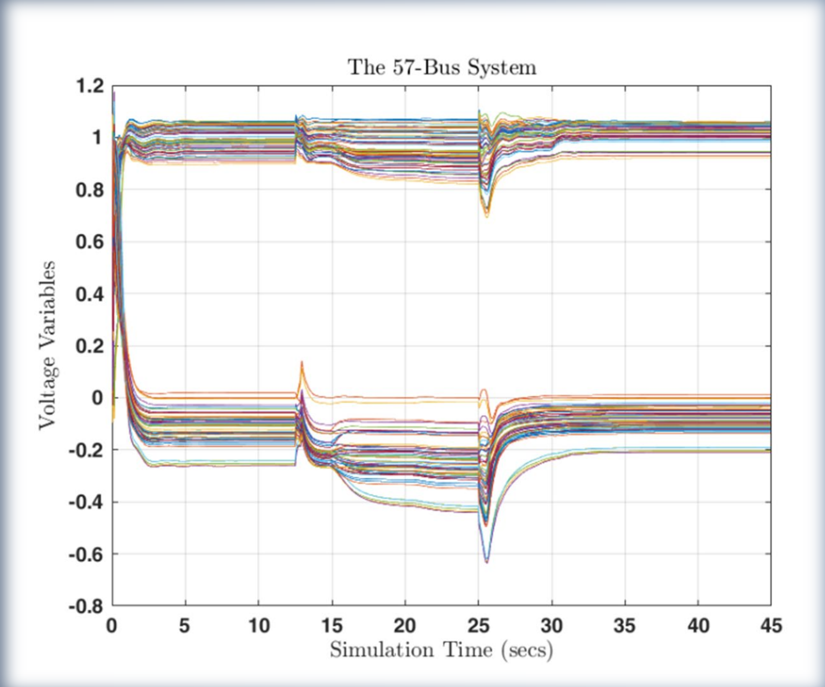
Efficiency of the proposed method when changing load

❖ In IEEE 57-bus when certain power system loads are increased to (active power: 50%, reactive power: 20%) at t = 6.8s and decreased to (active power: 50%, reactive power: 80%) at t = 13.6s, respect to nominal active and reactive loads.

The cost of power generation for the all generator in the 57-bussystem including variable loads at time 6.8 and 13.6 secs.



Trajectories of the voltage variables based on the 57-bus system including variable loads at time 6.8 and 13.6 secs.



Comparison of the best compromise solutions with other methods

- Comparison of the best compromise solutions of MOOPF problem for cases 1,2,3 for IEEE 14,30,39,57 and 118-bus

Case no.	Cost ($\$/h$)		Emission (ton/h)		Losses (MW)		CPU average time (s)	
	Proposed Method	Other Methods	Proposed Method	Other Methods	Proposed Method	Other Methods	Proposed Method	Other Methods
case 1								
14-bus	542		0.2235		*	*	2.6	
30-bus	846	830	0.1806	0.222	*	*	16.25	26
39-bus	32010		0.64		*	*	28	
57-bus	41800	42857	1.06	1.2	*	*	20.89	48
118-bus	87170		2.95		*	*	120	
case 2								
14-bus	181.8		*	*	1.325		1.98	
30-bus	810.4	804.96	*	*	2.65	3.85	15.5	27.58
39-bus	818.7		*	*	3.17		28	
57-bus	850		*	*	4.401		17.8	
118-bus	1647		*	*	9.43		70	
case 3								
14-bus	162.8		0.8267		2.39		1	
30-bus	820		0.211		3.131		6	
39-bus	828.2		0.6432		3.295		11	
57-bus	855.3		0.745		4.61		25	
118-bus	1873.1		1.65		8.766		180	